Course Title: Advanced Analysis

Course Code: MATH-307

Course Type: Major Math

Prerequisites: Real Analysis & Topology

Credit Hours: 3 (3 + 0)

Course Objectives: After completion of this course, the students will be able to:

- Understand and apply the concepts of Riemann integrals, Riemann-Stieltjes integrals and improper integrals.
- Study functions of bounded variation and their properties, including monotone functions and derivatives.
- Understand the concepts of Lebesgue measure, Lebesgue measurable functions and Lebsegue integration.

Course Contents:

Riemann-Stieltjes Integrals: Review of the Riemann integral, Definition of Riemann-Stieltjes integrals, Existence of upper and lower Riemann-Stieltjes integrals, Existence of Riemann-Stieltjes integrals, Properties of Riemann-Stieltjes integrals, Applications of Composition Rule, Riemann-Stieltjes integrals over a set of discontinuities, Relationship between Riemann integrals and Riemann-Stieltjes integrals, Fundamental theorem of calculus and its applications. Functions of Bounded Variation.

Improper Integrals: Points of infinite discontinuities and unbounded intervals. Types of improper integrals, Tests for convergence of improper integrals (Divergence, Comparison, Limit Comparison, Ratio, and Root tests), Beta and gamma functions and convergence of their integrals, Absolute and conditional convergence of improper integrals.

Lebesgue measurable sets: Sigma algebras and Borel sigma algebras, Definition and examples of Lebesgue measurable sets, definition of Lebesgue measure, Additivity of Lebesgue measure, Lebesgue measurable space, construction and Lebesgue measure of Cantor set, existence of non-Lebesgue measure sets.

Lebesgue measurable functions: Definition and examples of measurable functions, equivalent conditions for a measurable function, algebraic operations with measurable functions, Simple functions, Lebesgue measurable functions. Lebesgue Integration of simple functions and measureable function.

Recommended Books:

1. Axler, S., *Measure, Integration & Real Analysis*, Graduate Texts in Mathematics, Springer, 2020.
2. Bartle, G. R. and Sherbert, R. D., *Introduction to Real Analysis*, Wiley, 4th edition, 2011.

3. Gaskill, H. S. and Narayanaswami, P. P., *Elements of Real Analysis*, Prentice Hall, 1st edition, 1997.

4. Parzynski, W. R., *Introduction to Mathematical Analysis*, Mcgraw Hill College, 1st edition, 1983.

5. Rudin, W., *Principles of Mathematical Analysis*, McGraw-Hill Publishing Company, 3rd eEdition, 1976.

6. Tao, T., *An Introduction to Measure Theory*, American Mathematical Society, 2021.

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